Visualization of the viscous effects of non-Newtonian fluids flowing in mini-channels

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Abstract The velocity profile across a conduit is vital information for a number of applications. Here, the development of the velocity profile from the entrance regime to the developed regime was studied in detail specifically in for the case of a non-Newtonian fluid. The velocity profile across a rectangular mini-channel was measured using particle image velocimetry (PIV). The channels were made from poly methyl methacrylate (PMMA) and were fabricated from solid models using a commercial laser cutter. Polyacrylamide solution with shear-dependent viscosity (shear-thinning) was used as the working fluid and the fluid with different shear-dependent exponents (shear-thinning rate) was used to examine the change in the velocity profile. The instantaneous velocity profiles for different Reynolds numbers (Re) were compared with the theoretically derived velocity profiles to comment on the accuracy of the adopted measurement technique. Rheological measurements using a rheometer were preformed to obtain the power law for the non-Newtonian fluids at room temperature. In the case of non-Newtonian fluids, the instantaneous velocity profiles were acquired with the same flow rate as used for the Newtonian fluid case. It is evident that the velocity profile has strong dependency on fluid viscosity and the shear thinning characteristics. The aim of this work was to apply the PIV technique to obtain velocity profiles, which will be curve-fitted to obtain the rheological parameters *in-situ*. In this study it was observed that the rheological properties obtained from the velocity profile closely matches with the measurements performed using the rheometer. This method is a paradigm shift for the measurement of rheological properties of complex liquids which are difficult to characterize using conventional rheological measurement tools

Keywords: Flow profile, mini-channel flow, viscosity measurement, non-Newtonian flow, PIV

Introduction

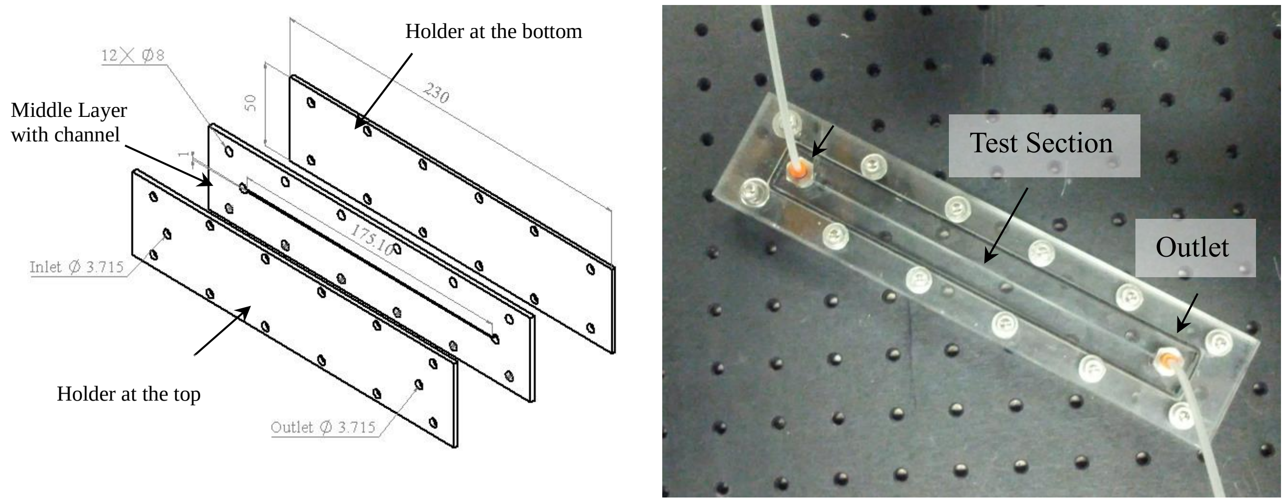
In microfluidic devices the high rate of mass and heat transfer as a result of large surface to volume ratio, makes them important in a variety of application such as; heat transfers, biomedical and chemical studies [1]. It is well document that the development velocity profile across the micro channels with non-Newtonian fluid does not follow the conventional wisdom [2-5]. In the case of Newtonian fluids the friction factor of different micro channels is determined based on the velocity profile data and it was reported that experimentally obtained data does not follow the theoretical prediction [2-10]. Very little attention has been given to this deviation between the experimental results and theoretical predictions particularly for the non-Newtonian fluids.

Exact quantification of viscosity is crucial parameter to dictate the energy needed to transport the liquid. It becomes further important in the case of micro channels where the surface forces are significantly larger than the body forces which in turn require very high pressure drop/pumping power to transport the fluid. Typically, only *ex-situ* viscosity measurements are performed with the help of rheometers. In such measurements the liquid sample is collected and measurements are carried out to obtain the bulk rheology of the fluid or the variation of the shear rate and shear stress. Among the studies which were used to model the relation between shear rate and shear stress, Ostwald-de Waele power law model is being used widely for Newtonian and non-Newtonian fluid determine the power law index of the fluid [1,11]. In several applications such *ex-situ* measurement is not feasible and hence *in-situ* viscosity measurement is required to obtain the rheological signature of the liquid.

In this study, the velocity profile across the microchannel with non-Newtonian fluid is experimentally obtained which further validated with the proposed theory and finally, a unique way to determine the rheological parameters from the velocity profile data is presented. The theoretical model and experimental model is validated against the Newtonian fluid. In order to validate the results, experimental rheological parameters of the polyacrylamide (non-Newtonian fluid) obtained from velocity profile information are compared with the results obtained from the parameters obtained from the rheometer.

Flow cell design

The flow cell with embedded microchannel used for the velocity profile analysis is shown in Fig 1. This flow cell consists of 3 layers as shown in Fig. 1(a); side walls for optical access are the top and bottom layers and the channel is cut into the middle layer. Layers were made from sheet of PMMA (Optix; Plaskolit Inc.) and a laser cutter (VersaLaser VLS Version 3.50; Universal Laser Systems) was used to obtain the required features. These three layers were fixed together with twelve #10-24 hex socket cap head screws and an o-ring was used to seal the layers as shown in Fig. 1(b). The top layer has inlet and outlet with diameter of 3.175 mm and 1/8" The length of channel in the middle layer was sufficiently long enough to avoid the entrance and exit effects hence enabling the analysis of the developed flow field across the field of view. To pump the fluid through the narrow channel with flow rates from 0.05 ml/min to 0.1 ml/min, a syringe pump (74900-00 Single Syringe Infusion Pump; Cole Parmer Instrument Company) and 5 mL syringe was used.

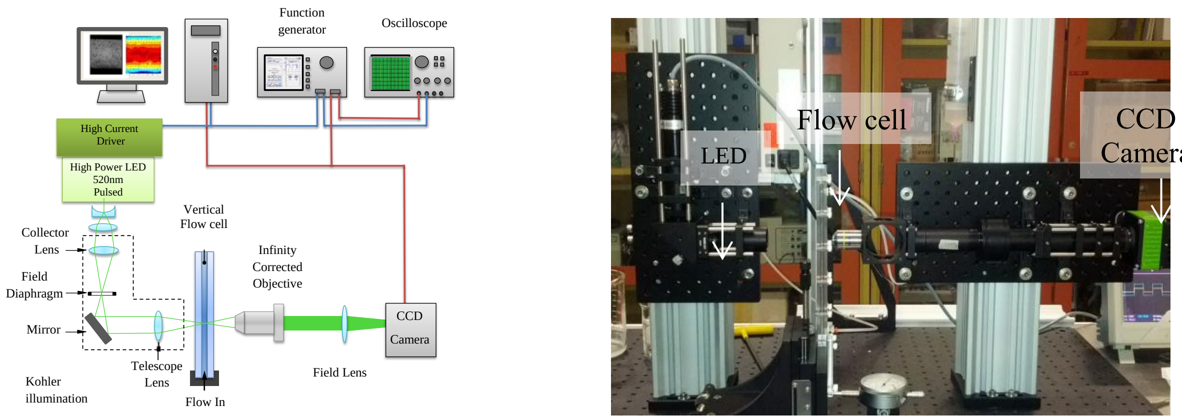


(a) (b)

Fig. 1. Flow cell used for analyzing the flow field across the microchannel (a) design of three layers of cell: top, bottom and middle. All dimensions provided in the image are in mm and (b) image of the flow cell with inlet and outlet.

Optical setup

The development of velocity profile was captured using a shadowgraph PIV setup as depicted in Fig. 2(a) and (b). To capture the sequence of images along the mini channel, a camera (SP-5000M-PMCL-CX; JAI Inc) with resolution of 2560 × 2048 pixels with 5× objective lens (PLN; Nikon) was used. A green high current LED light source (SL112-520 nm; Advanced Illumination Inc) was in pulsed mode to freeze the particle motion. A 2-channel function generator (TDS 2024B; Tektronix) allowed phase control of two continuous square waves at a frequency of 90 Hz to trigger the LED and camera. To have equally distributed intensity over the image, the distance of the lenses, LED and chip were calculated using a Kohler illumination [13] configuration. The picture of the physical setup is shown in Fig. 2(b). This highlights the arrangement and flexibility of the setup. Sample fluids within the mini-channel where seeded with polystyrene spheres particles with mean diameter of 1.0µm (R0100; Thermoscientific Company) to track fluid motion. The images captured where therefore a shadowgraph of the frozen particle field.

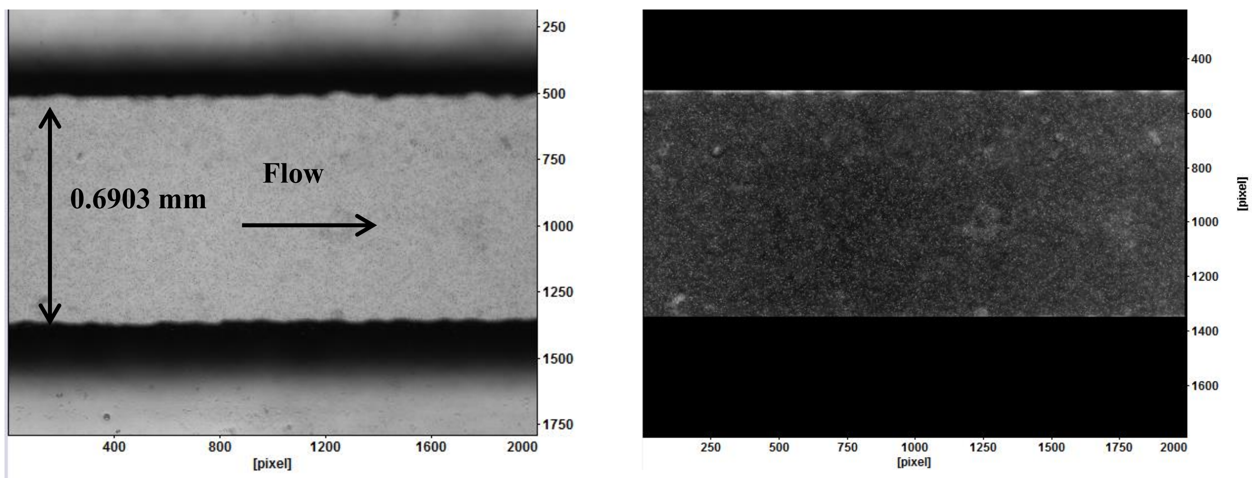


(a) (b)

Fig. 2. Optical setup used for obtaining velocity profile across the micro-channel (a) Schematic of PIV setup and (b) pictorial image of the experimental setup

PIV Processing

In order to obtain the velocity profile, a time series of 1,000 images collected at a frequency of 90 fps were collected for processing. The average of particle diameter in the image is around 1 to 2 pixel with displacement of around 4 to 5 pixels between sequential images. After calibrating the camera and converting the pixels in the image to mm, the width of channel was found to be 0.6903 mm. A commercially available PIV processing software (DaVis 8.1.4, LaVision GmbH) was used to process the images to velocity fields. Preprocess of the data was used to decrease the background noise and a geometric mask was applied to highlight only the flow domain. The captured image before and after preprocessing are shown in Fig. 3. The arrow in Fig. 4(a) indicates the direction of the flow. This image was inverted for PIV processing so that the particle images appeared bright as shown in. 4(b). The window size was selected to be bigger than the particle size for cross-correlations. Time series cross correlation PIV with a window size of 32 × 32and 75 % overlap was use for processing the images. The final velocity profile was obtained by averaging all 1,000 images.

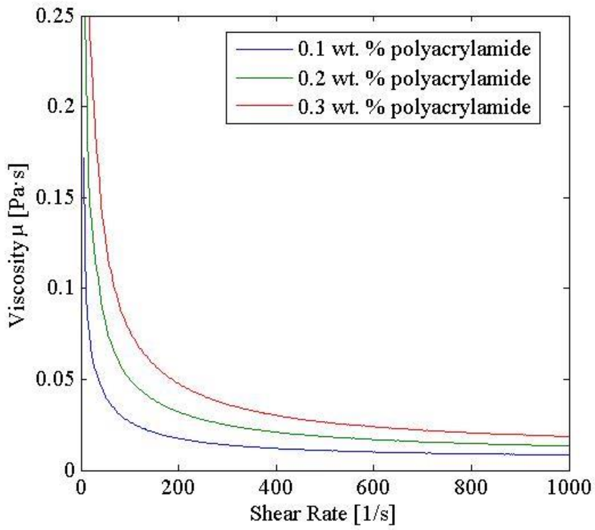


(a) (b)

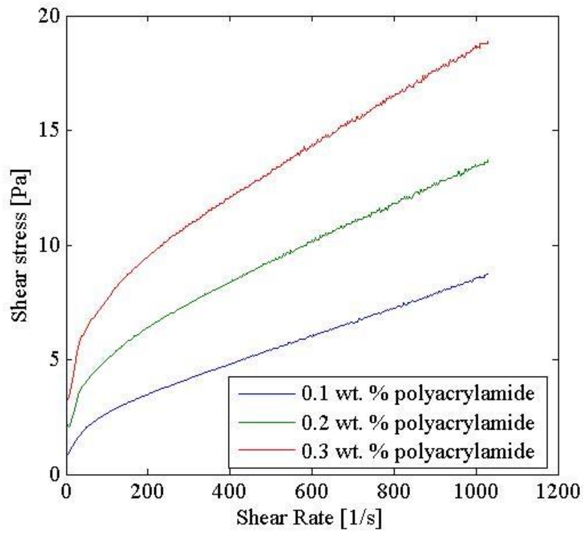
Fig. 3. Images captured using devised optical set up for obtaining the velocity profile across the microchannel (a) raw image captured by the camera and (b) inverted image with geometric mask for quantification of the velocity vector.

Solution preparation

Deionized water and 0.3 wt. % polyacrylamide/water solutions were used as the Newtonian and non – Newtonian fluid respectively to study the effect of fluid rheology on the velocity profile. To prepare polyacrylamide solution, a commercial sample of off-white granular powder high molecular weight anionic polyacrylamide powder (BASF SE, Germany) with particle size of 1,000µm and bulk density of 0.7 g/cm^3 of was used. To prepare solution with concentration of 0.3 wt. % of polyacrylamide (3000ppm), 0.3grams of polyacrylamide powder was gradually added to 100 ml of deionized water. To mix the solution a magnetic stirrer with the speed of 100 rpm was applied for 3 hours. A vacuum pump was used for degasification of the solution after mixing to decrease the potential of bubble formation in the mini-channel. The degasification time depends on the amount of bubbles in the solution and the degasification continued until the bubbles disappeared completely. The viscosity of polyacrylamide solution at different shear rate was measured using a rheometer [RheolabQC; Anton Paar] with double gap cylinder measuring system (DG42). Fig. 3 (a) shows the variation in viscosity of polyacrylamide solution (0.1 wt. %, 0.2 wt. % and 0.3 wt. %) with the change in the shear rate, whereas the Fig. 3 (b) depicts the variations in the shear stress for similar range if shear rate. To avoid any variations in the viscosity measurements due to the change in the room temperature the measurements were performed at constant 25°C.



(a)



(b)

Fig. 4. Rheological measurements of non-Newtonian fluid.(a) variations in the viscosity, and (b) shear stress with respect to change in the shear rate for different concentration of polyacrylamide at 25°C

According to the results presented in Fig. 3, the fluid has a shear thinning behavior and this behavior is dominant at lower shear rate. For non-Newtonian fluid the relation between shear rate and viscosity can be written as:

$$\tau = m\gamma^n \quad (1)$$

where τ is the shear stress, γ is shear rate and m is flow consistency index and n is flow index. The parameters of different samples considered in this study, following the Ostwald-de Waele power law model, are shown in Table 1.

Table 1. Rheological parameters for different concentration of polyacrylamide solution using the Ostwald -de Waele power law model

Solution concentration	Flow index (n)	m
0.1 wt.% Polyacrylamide	0.46888	0.28265
0.2wt.% Polyacrylamide	0.37755	0.91306
0.3 wt.% Polyacrylamide	0.27719	2.1691

Theoretical Background

The generalized representation of the non-Newtonian, power law fluid is presented in terms of the relationship between the shear rate and the corresponding shear stress with appropriate consistency index and power law index. Similarly the effective viscosity of the fluid can also be presented as:

$$\mu_{eff} = m\left(\frac{\partial u}{\partial y}\right)^{n-1} \tag{2}$$

Based on the power law index, the fluid can be differentiated into three different categories. The fluid is considered as a shear thickening fluid if the power law index is greater than one, for example a suspension of corn starch in water. Whereas, if this index is less than one the fluid is termed as a shear thinning fluid for example silicon oil. For the Newtonian fluid the flow consistency index is the effective viscosity.

From the conservation of momentum equation along the direction of fluid flow (x-direction) the velocity profile can be obtained for a power law fluid across the cross section of the channel as follows [14]:

$$u_{power} = \frac{n}{n+1}\left(\frac{1}{m}\frac{dp}{dx}\right)^{\frac{1}{n}}\left(y^{\frac{n+1}{n}} - H^{\frac{n+1}{n}}\right) \tag{3}$$

where H is the half of the channel width and dp/dx is the pressure drop along the channel. For $n = 1$, i.e., for a Newtonian fluid, the parabolic velocity profile expression can be obtained, commonly known as the Poiseuille flow equation. Fig. 4 shows the variation in the velocity profile for wide range of flow indices. The shear thinning nature ($n < 0$) shows a flat peak at the center of the channel due to sudden change in the shear rate whereas in the case of shear thickening fluid the shear rate diffuses slowly. The Newtonian fluid ($n = 1$) shows a perfect parabola. From this it can be seen that the nature of the fluid (whether it is shear thinning or thickening) can be determined by carefully observing and quantifying the velocity profile across the channel. To support this hypothesis, velocity profile across the channel for different power law fluids is obtained.

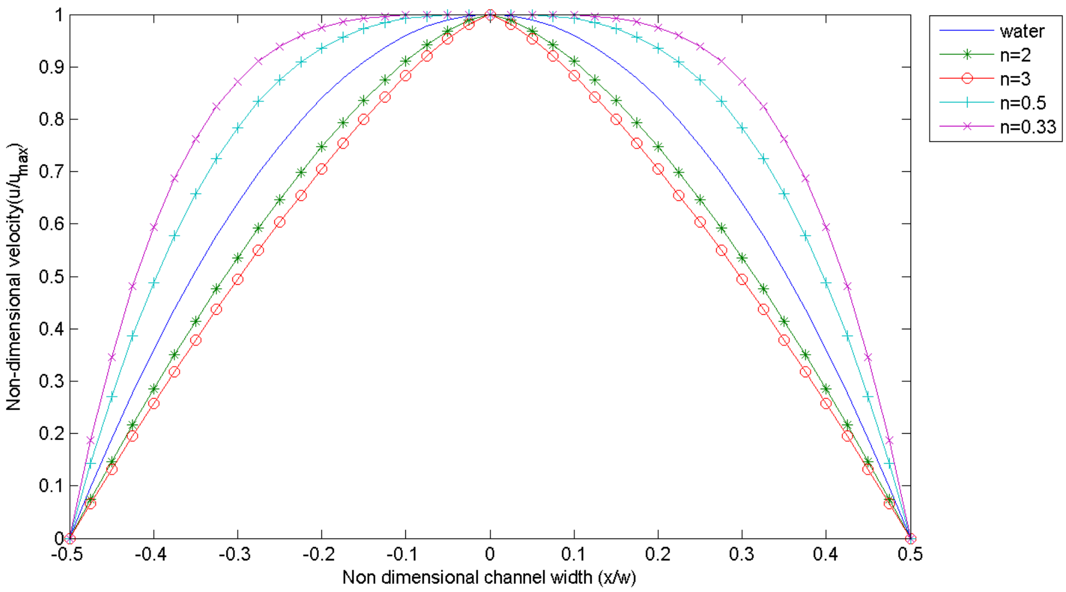


Fig. 5.Fully developed velocity profile for different power law fluids

Velocity profile of Newtonian and non-Newtonian fluid through narrow straight channel

The developments of velocity profile of both Newtonian and non-Newtonian fluid were investigated at flow rate of 0.05, 0.075 and 0.1 ml/min to determine the effect of flow rate on the fluid on velocity profile. The average velocity vector field mapped onto a colour map of the velocity magnitude of water through the micro-channel is shown in Fig. 6. Every 5th vector is shown highlighting the general flow characteristics. The velocity was normalized with maximum velocity through the channel to compare the result of different cases. The velocity field has a parabolic profile as it was expected for Newtonian fluid. To validate the experimental results, the obtained velocity profiles for water are compared with theory presented in the *Theoretical Background* section. Fig. 7 shows the velocity profile for water with three flow rates, i.e., for 0.05 ml.min^{-1} [Fig. 7 (a)], 0.075 ml.min^{-1} [Fig. 7 (b)] and 0.1 ml.min^{-1}[Fig. 7 (c)] As it is shown in Figs.7 (a) to (c), the experimental velocity profile of Newtonian fluid matches theory perfectly at three different flow rates. It can be conclude that the experimental approach captured the appropriate of the flow.

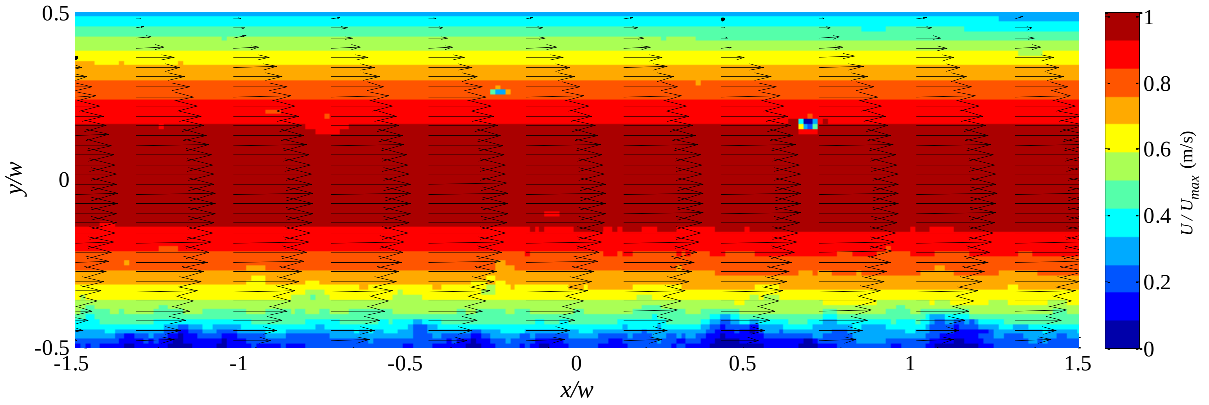
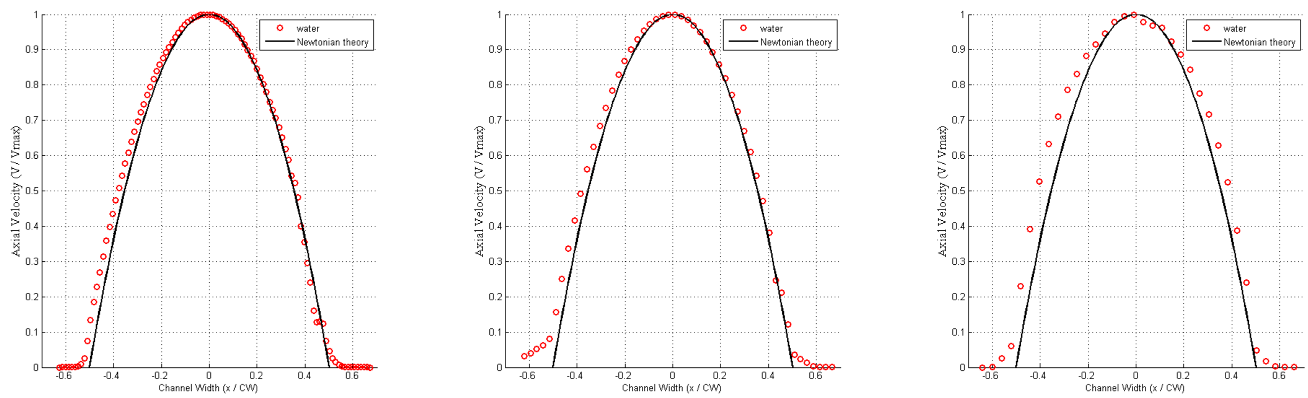


Fig. 6. Average velocity profile vector map (only every 20th vector in *x*-dir shown) with a background colour map of velocity magnitude for water through 1mm straight channel at flow rate of 0.1 ml.min^{-1}



(a) (b) (c)

Fig. 7. Comparison of average velocity profile of water at (a) 0.05 ml.min^{-1} (b) 0.075 ml.min^{-1} and (c) 0.1 ml.min^{-1} with theory

To study the effect of the fluid rheology on the velocity profile, 0.3 weight % polyacrylamide was used as the working fluid. The development of the velocity field for this non-Newtonian fluid along the channel with 0.1 ml/min flow rate is shown in Fig. 88. The velocity profile of the polyacrylamide is not parabolic as a result of shear thinning behavior of the solution. The velocity profiles of polyacrylamide solution for four different flow rates with three different polyacrylamide solutions are presented in Fig. 9. It is observed that the velocity field of polyacrylamide solution at different flow rates (0.025 ml/min, 0.05 ml/min, 0.075

ml/min, 0.1 ml/min) has a flatter profile compared with the Newtonian fluid which is popularly termed as plug flow profile. Three different solutions are also compared for same flow rate Fig. 9 (a)-0.1%, Fig. 9 (b)-0.3% and Fig. 9 (c)-0.5% and it is evident that in all three cases the centerline velocity increases as the flow rate increases. As the polyacrylamide concentration increases, due to the increment in the shear thinning nature of liquid, the profile changes from parabolic in shape to plug-like shape. The strength of the shear thinning or flow index dictates the magnitude of the width of the flattened profile. For a given polyacrylamide solution the flow index is constant parameter hence the width of flattened profile is same for different flow rate but as the polyacrylamide concentration is increased the width of the flattened profile significantly changes. In the case 0.1% solution the flattened profile area is negligible compared to the width of the channel whereas in the case of 0.3% and 0.5% it is ~ 40% and ~ 60% of the width of the channel.

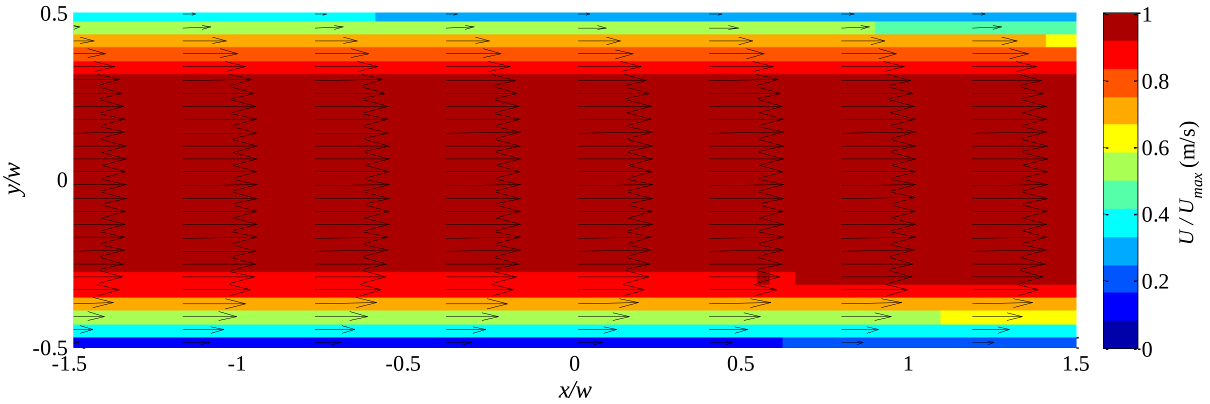
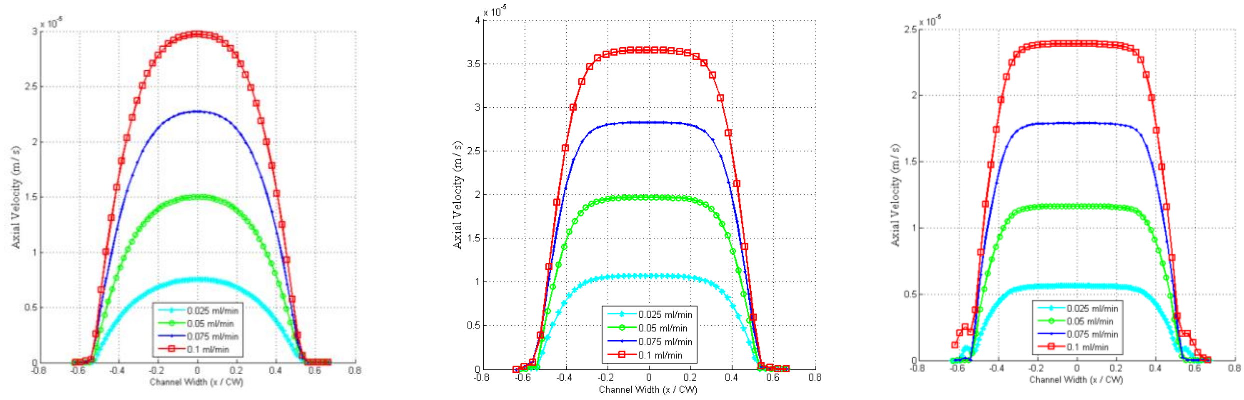


Fig. 8. Average velocity profile vector map (only every 20th vector shown) with a background colour map of velocity magnitude for 0.3% polyacrylamide through 1mm straight channel at flow rate of 0.1 ml.min^{-1}



(a) (b) (c)

Fig. 9. Velocity field of (a) 0.1 wt. % polyacrylamide solution (b) 0.3 wt. % polyacrylamide solution (c) 0.1 wt. % polyacrylamide solution at different flow rates (0.025ml/min, 0.05ml/min, 0.075ml/min, 0.1ml/min)

To find the rheology of the polyacrylamide solution, knowledge of flow index required. The flow index can be obtained by comparing the theoretical outcome with the experimentally observed velocity profile data. Figure 10 depicted a sample curve fitted plot for 0.3 wt. % polyacrylamide solutions at flow rate of 0.05ml.min^{-1}. Here, the experimentally obtained non-Newtonian normalized velocity profiles were fitted with a power law function. After comparing the exponents of the curve fitted equations with the power law function fluid indices of different polyacrylamide are determined as shown in Table 2. The average of calculated flow indices of 0.3 wt. % polyacrylamide found to be 0.2812. Thus one can determine the rheological properties from the velocity profile information. To validate the accuracy of the obtained rheological parameters, the flow indices determined from the experimentally obtained velocity profile are compared with the indices obtained from rheometer measurements. It is to be noted that the indices obtained from the velocity profile data are function of the flow rate but the variations in the magnitude of these indices are within the tolerance of the instrument. Hence one can consider these variations are negligible. Finally, the experimentally observed velocity profiles are compared with the theoretical analysis as depicted in Fig. 11. For the theoretical analysis the rheological parameters obtained from the curve fitted analysis are used. The velocity profiles for three different flow rates [Fig. 11(a) - 0.05 ml.min^{-1}, Fig. 11(b) - 0.075 ml.min^{-1} and Fig. 11(c) -0.1 ml.min^{-1}] show good agreement with the theory. Here, the velocity profile of non-Newtonian fluid was obtained based on the Ostwald-de Waele power law theory.

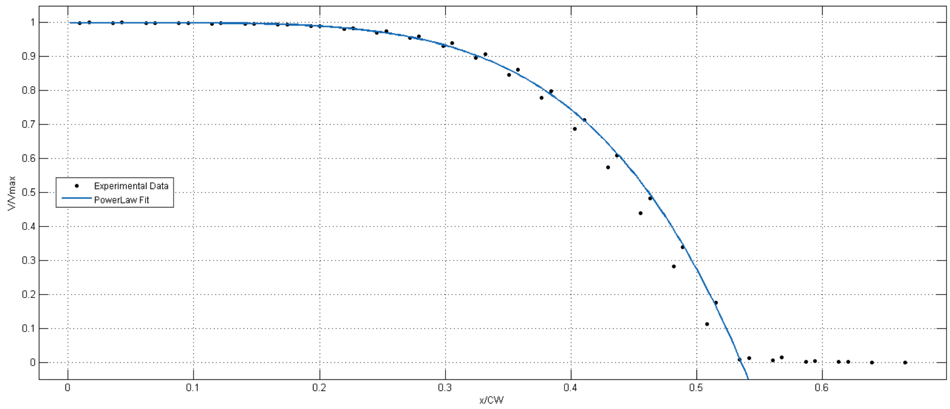
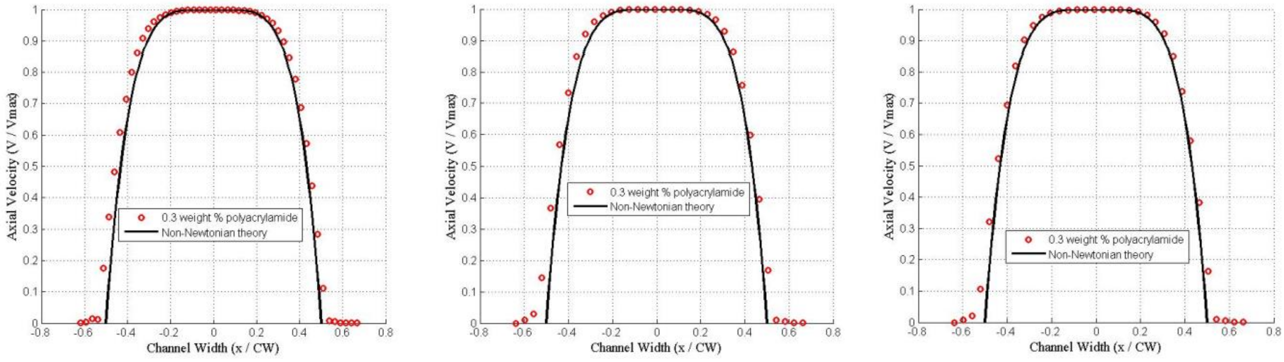


Fig. 10. Curve-fitted plot for 0.3 wt. % polyacrylamide solution at flow rate of 0.05 ml.min^{-1}

Table2- Flow behavior indices calculated after curve-fitting experimental velocity profile for 0.3 wt. % polyacrylamide

Flow rate	Values of *n* obtained from curve-fitting shown in10
0.05 ml.min^{-1}	0.2726
0.075 ml.min^{-1}	0.2971
0.1 ml.min^{-1}	0.2739



(a) (b) (c)

Fig. 11. Average velocities of 0.3 wt. % Polyacrylamide at (a) 0.05 ml.min^{-1} (b) 0.075 ml.min^{-1} and (c) 0.1 ml.min^{-1} fitted with the theoretical non-Newtonian equation (obtained after substituting values of *n*).

Conclusion

The development of velocity profile of Newtonian and non-Newtonian fluid across the microchannel was studied using PIV method. In this experiment DI water and 0.3 wt. % polyacrylamide solution were used as Newtonian fluid as non-Newtonian fluid, respectively. According to the study of the Newtonian flow field through micro-channel, the velocity has parabolic profile which shows good agreement with the theory. The velocity profile for three different non-Newtonian fluids with different flow rates is obtained. Further, the flow index of polyacrylamide was determined form these experimentally observed velocity profile. The obtained flow indices, i.e. rheological properties of the liquid solution were compared with the properties measured from the rheometer. It was observed that the rheological properties achieved from the velocity profile closely matches with the measurements performed using the rheometer. The theoretical profile was plotted using the calculated flow index and the results reveal that in case of shear thinning fluid, the curvature of the velocity profile is smoother than the parabolic velocity profile of Newtonian. It has been demonstrated that the rheology of the fluid can be characterized with the knowledge of the velocity profile.

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