Nonlinear shape evolution of immiscible two-phase interface

Francesco Capuano[1,2,*], Gennaro Coppola[1], Luigi de Luca[1]

[1]Dipartimento di Ingegneria Industriale (DII), Università di Napoli "Federico II", Napoli, 80125, Italy
[2]Centro Italiano Ricerche Aerospaziali (CIRA), Capua, 81043, Italy
[*]corresponding author: f.capuano@cira.it

Abstract The Kelvin-Helmholtz (KH) instability is one of the most elementary and widespread models of fluid dynamics. The model is successful in predicting several physical situations, although in its basic formulation it does not consider the finite thickness of the shear layer and the nonlinear saturation of the exponential amplification of disturbances. However, there are flow configurations, such as those of fuel injection systems, where the basic KH linear model fails because the nonlinear effects play a major role in the process of transition from stratified to slug flow, and therefore the finite amplitude of the instability wave has to be taken into account starting from the early instants. Previous contributions of literature are due to Orazzo et al. [1], for non-parallel channel shear flow, and to Hoepffner et al. [2], for unconfined flows and disturbance wave produced by a localized impulse force. In both cases, the relevant characteristic of the inherently nonlinear instability is the emergence of a single travelling wave, whose characteristic velocity is different from the one of the classic linear KH theory.
In the present work, visualizations of a nonlinear Kelvin-Helmoltz instability is obtained via direct numerical simulations of the Navier-Stokes equations for a gas-liquid flow confined in a channel. Simulations are performed by means of the interFoam solver, based on the volume of fluid (VOF) method. The code is included within the open-source package OpenFOAM and is widely validated for the flows of interest [3]. The different features that distinguish the single-wave scenario from the classical train of KH linear waves are highlighted and discussed in the paper.
Keywords: Two-phase channel flow, Kelvin-Helmholtz instability, Volume of Fluid

1 Introduction

The Kelvin-Helmholtz (KH) instability is one of the most simple and widespread (linear) models employed in fluid dynamics [4]. In its basic formulation it usually refers to the configuration of two irrotational (uniform) fluid currents flowing on each other, these being separated by a vortex sheet (line) whose instability is analyzed. The classical approach employed for the analysis of this problem is that of the study of the linear evolution of a small periodic perturbation superposed to the undisturbed separating surface; this approach extending also to the case of two immiscible fluids. As is well known, the vortex sheet simulates a zero-thickness mixing layer, and this is the reason why its instability is generally attributed to shear, although, as pointed out by [5], it is produced by the action of the pressure. In spite of its over-simplicity, the model appears to be successful in predicting a wide set of physical situations ranging from the motion of billow clouds to the propagation of waves generated by the wind on the sea.

A different configuration, which is actually the one originally considered by Helmholtz, is that of a localized perturbation of the interface between two parallel streams of immiscible fluids. This problem has received less attention than the classical wavelike periodic perturbation approach, mainly because of the mathematical difficulties related to the inherently nonlinear character of the evolution of the localized perturbation.

The case of two-phase gas-liquid flow confined in a channel is a typical situation in which the classical KH linear model fails; the major reason for the disagreement between theory and experiments is that non-linear effects have a major importance on the process of transition from stratified to slug flow, and therefore the finite amplitude of the instability wave has to be taken into account starting from the early instants.

In a recent paper ([1]), the emergence and the subsequent amplification of a non-linear single-wave in non-parallel channel flow is analyzed, both theoretically and numerically. The aim of the present contribution is to justify more in depth some features of the single wave by employing numerical flow visualization as a tool for the modelling of the wave dynamics.

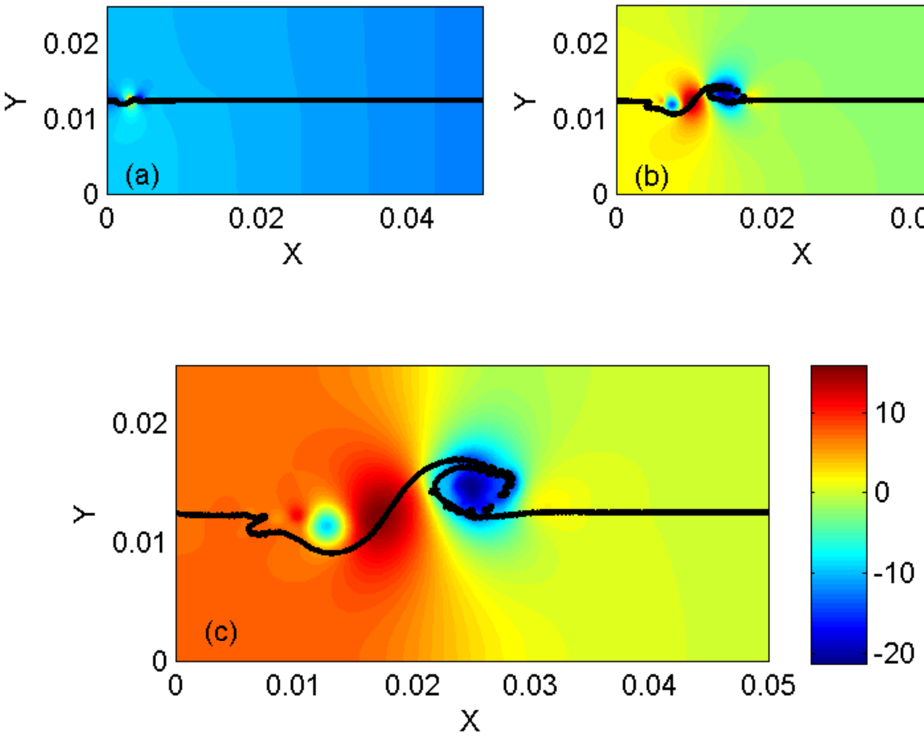

Fig. 1 Spatial distribution of relative pressure at three different times during the evolution of the wave. (a) $t = 2 \times 10^{-3}$ s, (b) $t = 6 \times 10^{-3}$ s, (c) $t = 10^{-2}$ s. $U_l = 0.5$ m/s, $U_g = 6$ m/s.

2 Problem formulation and numerical simulation

The non-linear wave under study can be generated by considering the flow of two superposed fluids with different velocities, densities and viscosities entering a channel. Outside the chamber the two fluids are separated by a rigid splitting boundary; at the channel entrance the parallel streams merge, forming a two-phase shear flow. The sudden change of boundary interface condition causes the emergence of a finite amplitude single wave, which propagates as a growing perturbation of the interface between the two fluids. The flow regime considered here is that of incompressible, immiscible and Newtonian fluids. In what follows p, ρ, μ and U are pressure, density dinamic viscosity and velocity, respectively. The subscripts g and l refer to gas and liquid phases. The liquid occupies the lower part of the channel, of height h. The gas flows in the upper part, extending up to the channel height H. Gravity is not included into the analysis.

The model proposed in the next section for the dynamics of the evolving single wave conducts to the prediction of some quantitative features that are corroborated by numerical simulations. The solver adopted for the simulations is interFoam, which is part of the suite of C++ libraries of the open-source software OpenFOAM. The incompressible flow equations are solved by employing a finite-volume discretization on a collocated grid arrangement. Convective fluxes are reconstructed by a second-order centered scheme, while time integration is carried out by a two-step second-order method. Two-phase modeling is achieved through the Volume of Fluid (VOF) method, in which an evolution equation for a volume fraction variable is coupled to the Navier-Stokes equations. The face values for the volume fraction are calculated using a blended second-order interpolation, with the van Leer limiter being used to preserve boundedness. The solution of the equations in interFoam is performed by constructing a predicted velocity field and then correcting it using the Pressure Implicit with Splitting of Operators (PISO) implicit pressure correction procedure to time advance the pressure and velocity fields. A thorough validation of the software, as well as further details of the solution algorithm are given in ([3]). Boundary conditions are imposed as velocity inlet and pressure outlet for the simulation of the nonlinear

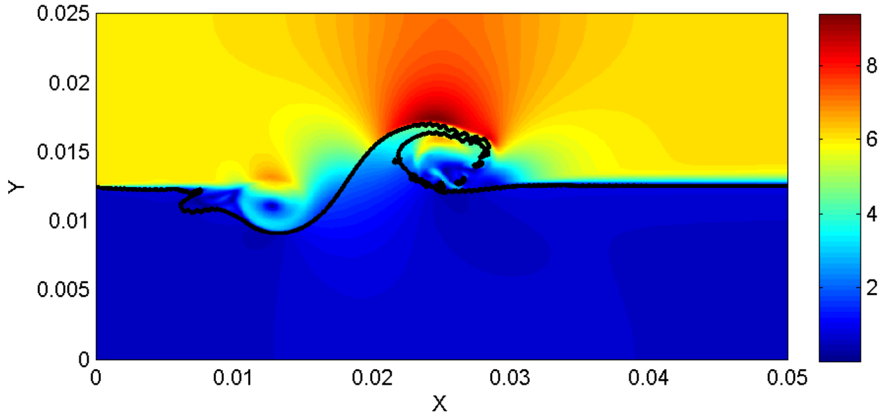

Fig. 2 Colormap of the modulus of the velocity. $t = 10^{-2}$ s.

wave. A free-slip, zero-normal-flux boundary condition is enforced at the channel walls. The simulation considered in this paper refers to a typical case of gas-liquid system, for which dimensional velocities, densities and viscosities are fixed in such a way that Reynolds numbers based on the height of the gas/liquid region are $\mathrm{Re}_g = 5 \times 10^3$ and $\mathrm{Re}_l = 76$ for gas and liquid phases respectively. Density and viscosity ratios are set to the values $\rho_g/\rho_l = 0.1$ and $\mu_g/\mu_l = 0.018$. Weber number is defined as: $\mathrm{We} = \rho_g U_g^2 h_g/\sigma$ where σ is the gas-liquid surface tension coefficient; its value is fixed to $\mathrm{We} = 10^5$.

In Fig. 1(a)-(c) three colormaps of spatial distribution of relative pressure are shown, at three different times during the evolution of the wave. The interface between gas and liquid is also visible as a thick black line. The Figures show that an high pressure region develops behind the isolated corrugation of the interface, connected to the low pressure region ahead it through a steep gradient which travels with the wave.

In order to physically describe the mechanisms of propagation of the wave, some other useful field quantities can be visualized, as an aid to the application of basic physical principles. Fig. 2 shows the colormap of the modulus of the velocity inside the domain for the time instant relative to Fig. 1(c). This visualization shows a region of high velocity above the crest of the wave, which corresponds to the low pressure region visible at the same location in Fig. 1(c). A high velocity region is visible also inside the wave, in the liquid phase. This is an indication that a suction velocity is present inside the wave, which sustains the wave and permits it to grow during the evolution.

3 A model for the wave dynamics

In regards to the basic characteristics of the single wave and of its dynamics, a model can be proposed based on the evaluation of pressure and velocity fields and on the application of simple dynamic principles. The propagation velocity of the wave can be estimated by means of a very simple model. The first step is the visualization of the dynamical properties of the gas and liquid regions near the crest of the wave in a reference frame that moves with it. Fig. 3 shows the streamlines in this reference frame, together with a colormap of the total pressure, made of the sum of kinetic energy and static pressure p. The different densities of the two phases are consistently taken into account in the calculation of kinetic energy. The Figure shows the location of a stagnation point behind the crest of the wave, at which the total pressures on the two sides of the interface are approximatively equal. The balance of forces normal to the interface at the stagnation point can hence be written as:

$$p_g + \frac{1}{2}\rho_g (U_g - c)^2 = p_l + \frac{1}{2}\rho_l (U_l - c)^2 \tag{1}$$

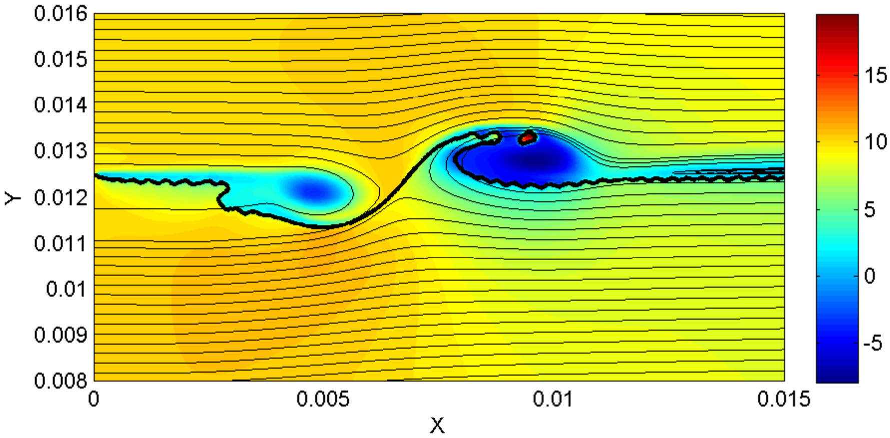

Fig. 3 Colormap of the total pressure $p + \frac{1}{2}\rho V^2$ and streamlines in the travelling reference frame.

where c represents the convective velocity of the stagnation point. Assuming $U_l < c < U_g$ and imposing local normal stress balance, $p_g = p_l$ (the effect of the surface tension can be neglected beacuse the curvature is nearly null at the stagnation point, at least in the early instants), yields

$$c = \frac{U_l + U_g\sqrt{d}}{1+\sqrt{d}} \tag{2}$$

where d is the density ratio. This expression for c has been derived in a different context as the propagation velocity of a vortex appearing in the mixing layer of two currents having different velocities ([8]).

In order to validate the prediction of the proposed model, an automatic tracking of different points characterizing the wave has been implemented. Fig. 4 shows a typical situation in which three points are located, corresponding to the maximum and minimum height of the wave (points B and C respectively) and to the point of intersection between the rear part of the wave and the position of the undisturbed interface (point A). The positions of these three points are recordered as functions of time during the evolution of the wave. Fig. 5 shows the space-time diagram of the streamwise coordinate of the three points, together with the prediction of formula (2). As it is clear from the figure, the agreement between the theoretical prediction and the numerically

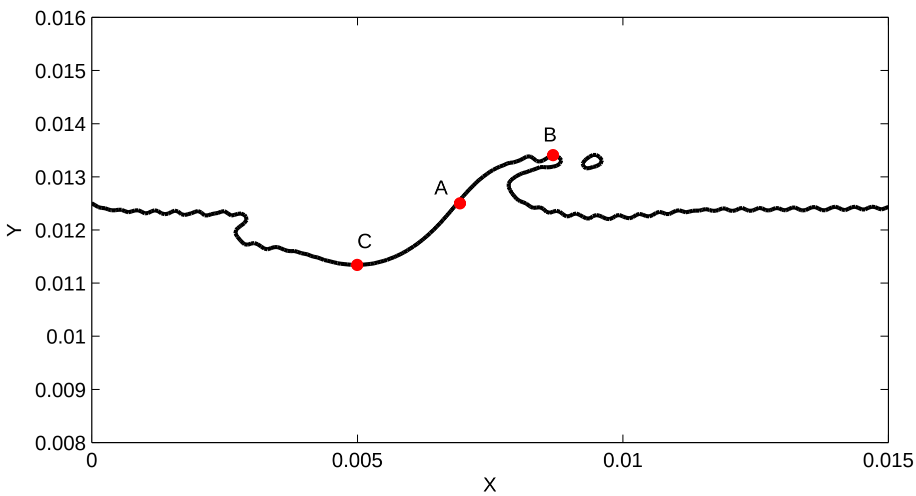

Fig. 4 Definition of the points A, B and C automatically tracked during the evolution of the wave.

calculated velocity of the wave is completely satisfying for the case of the point A, which is the location of the stagnation point. The different behaviour of the points indicating the maximum and minimum vertical wave extension shows that the wave is growing in its longitudinal elongation. Fig. 6(a) reports the temporal evolution of maximum and minimum heights of the wave as functions of time. In order to study the global growth of the wave, the differences between ordinates and abscissas of points B and C is also reported in Fig. 6(b). The Figure shows a linear growth of the wave in time, in agreement with previous analyses ([2]).

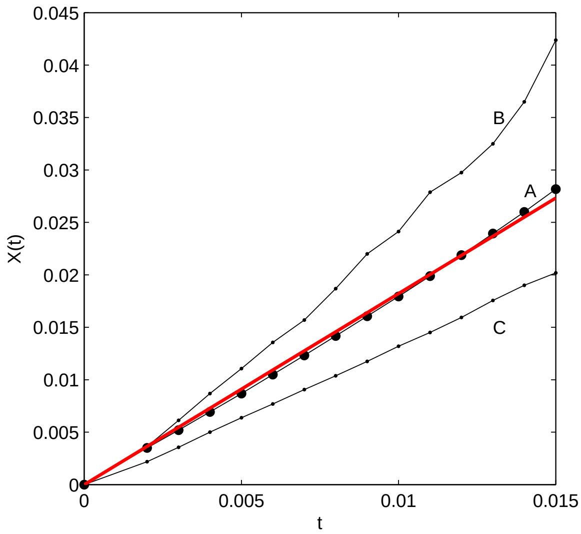

Fig. 5 Space-time diagram of the streamwise coordinate of the points A, B and C of the wave. The thick red line is the prediction of formula (2).

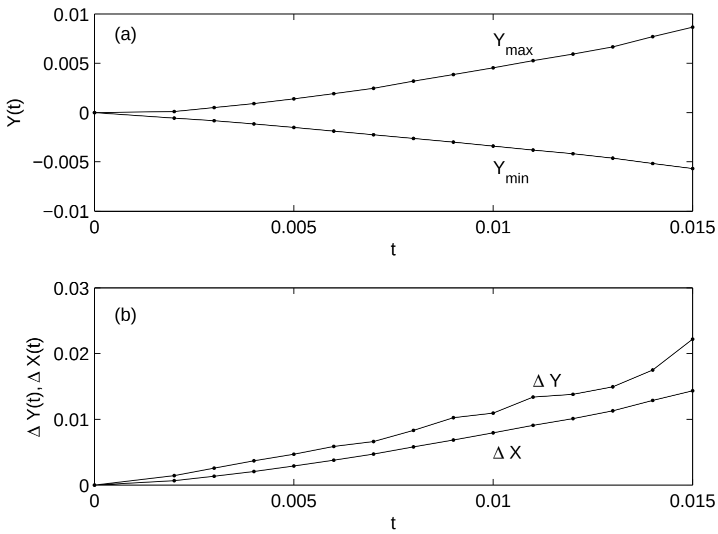

Fig. 6 Temporal evolution of characteristic geometric properties of the wave. (a) maximum and minimum heights of the wave. (b) Differences between ordinates and abscissas of points B and C of the wave.

4 Conclusions

We presented a numerical simulation of a nonlinear single wave occurring at the interface of two viscous and immiscible fluids when they interact in a chamber coming from two separate and parallel streams. The visualization of pressure and kinetic energy fields, coming from the numerical simulation, shows that some dynamical properties of the wave can be characterized by means of a simple inviscid model. The propagation velocity and the growing size of the wave have been numerically characterized and compared to the theoretical predictions of the simple model.

References

[1] Orazzo A, Coppola G, de Luca L (2011) Single-wave Kelvin-Helmholtz instability in nonparallel channel flow. *Atomization and Sprays*, vol. 21, pp 775-785. doi: 10.1615/AtomizSpr.2012004148

[2] Hœpffner J, Blumenthal R, Zaleski S (2011) Self-similar wave produced by local perturbation of the Kelvin-Helmholtz shear-layer instability. *Physical Review Letters*, vol. 106 pp 104502-1 - 104502-4. doi: 10.1103/PhysRevLett.106.104502

[3] Jasak H (2014) OpenFOAM: Open source CFD in research and industry *International Journal of Naval Architecture and Ocean Engineering* vol. 1, pp 89-94. doi: 10.2478/ijnaoe-2013-0011

[4] Drazin P G (2002) Introduction to Hydrodynamics Stability. Cambridge University Press

[5] Joseph D, Funada T, Wang J (2007) Potential flows of viscous and viscoelastic liquids. Cambridge University Press

[6] Varga C M, Lasheras J C, Hopfinger E J (2003) Initial breakup of a small-diameter liquid jet by a high-speed gas stream *Journal of Fluid Mechanics* vol. 497, pp 405-434. doi: 10.1017/S0022112003006724

[7] Funada T, Joseph D (2001) Viscous Potential Flow analysis of Kelvin-Helmholtz instability in a channel *Journal of Fluid Mechanics* vol. 445, pp 263-283. doi: 10.1017/S0022112001005572

[8] Dimotakis P E (1986) Two-Dimensional Shear-Layer Entrainment *AIAA Journal* vol. 24, pp 1791-1796. doi: 10.2514/3.9525