Particle Image Characteristics in Digital Holographic Particle Image Velocimetry and their Application in Three Dimensional High Speed Flow Measurements

Christina Hesseling¹⁺, Tim Homeyer¹, Joachim Peinke¹, Gerd Gülker¹

¹Institute of Physics, ForWind - Center for Wind Energy Research, Carl von Ossietzky University, Oldenburg, Germany
⁺Corresponding author: christina.hesseling@uni-oldenburg.de

Abstract Digital holographic particle holograms are recorded with an in-line setup and reconstructed by the convolution approach. From the reconstructed complex volume, the original particle positions are to be retrieved. A detailed analysis - experimentally as well as numerically - shows a strong effect of the particle position within the volume on the shape of the respective reconstructed particle image. In the experimental system applied here, microparticles of diameter below 9 µm are used, which is smaller than the employed camera pixel pitch of 12 µm. In this case, not only the distance of the particle position to a sensor edge is important but also its relative position to the camera pixel grid. It is demonstrated that a reconstructed intensity of a numerically simulated particle can vary by more than 50% even when the simulated particle is moved by no more than a particle diameter. This is a problem for particle detection algorithms relying only on an intensity threshold for particle detection. Furthermore, the cut-off of particle holograms on the edges of the camera sensor affects the reconstructed images. Therefore, an algorithm is introduced, which uses simulated particle images of the approximated transverse position which are correlated with the reconstructed experimental data to retrieve three dimensional particle positions.

Keywords: Digital Holographic PIV, Holographic Particle Images, Water Flow Measurements

1 Introduction

In all fields of fluid dynamics complex flow situations require three dimensional high speed measurements. One common technique is tomographic particle image velocimetry (PIV), employing usually at least four cameras and pulsed laser light. The idea of in-line holographic PIV, using only a single camera and continuous wave laser light of comparatively low intensity, is very appealing for several reasons. The light intensity is used very efficiently due to the strong forward scattering. Thus, the temporal resolution is limited by the framerate of the camera. The desired three dimensional information is directly recorded. The low light intensity and the single camera in use make the system simple and cost-efficient. Furthermore, low light intensity is favourable in some applications, e.g. with living organisms. In addition to the higher temporal resolution in comparison to analogue holography, recording of digital holograms favours digital reconstruction, which does not only provide information about the intensity but also about the phase of the object light field.

In spite of its advantages mentioned above, digital holographic particle image velocimetry has not yet become a standard technique. As digital sensors yield a low numerical aperture of the recording system, a large depth-of-focus can be observed in the intensity field surrounding each reconstructed particle position. Furthermore, random interference structures generate speckle in the intensity field [1], which can be misinterpreted as particles resulting in ghost particles in the retrieved particle set. To overcome these drawbacks, the work presented here does not only rely on the reconstructed intensity field for particle detection and validation but also employs the phase information. So the complete information of the reconstructed complex field is used. For this purpose, a study of the phase and the intensity field surrounding a reconstructed particle is presented.

The depth of focus problem has been addressed by many authors, e.g. by [2], [3] and [4], or, with a more sophisticated setup using multiple cameras by [5]. Not so much attention has been paid to the effect of the transverse particle location on the reconstructed particle image when the common convolution approach [6] for particle image reconstruction from a hologram is used. A short illustration of the truncation on the sensor edges affecting the reconstructed images can be found in [7]. In order to develop a particle detection algorithm which is capable of measuring particles in all areas of the sensor, it has to be known, what kind of particle images are to be expected from the system. In the course of this work, it becomes clear that the effect of the particle position in the plane perpendicular to the optical axis is very prominent.

The analysis of particle images in sections 4, 5 and 6 is followed by first results of the developed particle detection algorithm in section 7.
2 The In-line Digital Holographic Particle Image Velocimetry Setup

The digital holographic PIV setup is shown in figure 2.1. The water flow under investigation is induced by a rotating circular stirrer. The stirrer dips into the water filling a glass cuvette with a squared base of \((50 \pm 0.5)\) mm inner length. Polystyrene microparticles with an average diameter of \((8.69 \pm 0.12)\) \(\mu\)m are added to the water. These particles are illuminated by expanded and collimated laser light of a continuous wave Nd:YAG laser. Part of the illuminating laser light is scattered by the particles generating the object wave. The remaining part of the illumination beam acts as reference wave. Object and reference wave interfere on the sensor of a CMOS highspeed camera, which records the holograms with a framerate of 620Hz.

![Digital holographic PIV setup](image)

Fig. 2.1 Digital holographic PIV setup for measurements of a water flow induced by a stirrer in a glass cuvette with squared base of \((5cm \pm 0.5)\) cm inner length.

The extension of the camera sensor is used as optical low pass, as described in [8]. The Nyquist limit of the camera sensor is defined by its pixel pitch. On the other hand, the spatial frequency of the diffraction structure caused by a particle increases with increasing angle between the object and the reference wave. This can be seen in figure 2.3, in which the circular interference patterns of particles are visible. With increasing distance between scattering centre and recording plane, the total number of fringes reaching the sensor decreases. Hence, the camera sensor is positioned sufficiently far from the scattering particles, that in the horizontal and vertical extension of the region of interest of the hologram no spatial frequency is recorded which exceeds the Nyquist limit. For the system parameters used here, the minimum distance between camera sensor and scattering particles is therefore set to 27.7 cm.

From this minimum distance and the extension of the sensor, the largest aperture half angle \(\Omega\) is deduced for this system. The depth of focus of a reconstructed particle can be estimated from this aperture half angle by [9]

\[
\Delta z = \frac{\lambda}{\Omega^2}.
\]  

Hence, here the approximated depth of focus of a particle reconstructed in the distance of 27.7 cm to the camera sensor on the optical axis is more than 1 mm, which is about 100 times the diameter of the employed particles.

A hologram recorded by this system can be seen in figure 2.2. Strong interference fringes are visible which are not caused by the microparticles but which are due to unwanted reflections at temporally constant structures in the setup, like the protective window of the camera sensor. In the measurements only the temporally moving interference structures caused by the tracer particles are of interest. Therefore, the holograms are preprocessed before they are reconstructed by subtracting a temporal mean of several holograms from each hologram with the appropriate normalization. The result of a preprocessing routine can be seen in figure 2.3. Here, the temporal mean of the previous and the following three holograms in the sequence was used for the preprocessing. The circular interference fringes expected from microparticles become clearly enhanced by the preprocessing while the central large circular pattern and the approximately vertical lines are hardly visible any more. Because
structures moving in time remain in the filtered hologram, a great effort is made to keep the system as spatially stable as possible. Still, fluctuations, for example due to the camera fan, cannot be completely avoided.

3 Hologram Reconstruction by the Convolution Approach

The complex light field is reconstructed from the filtered holograms by the well-known convolution approach [6] with the Fresnel-Kirchhoff formula [10]. For simplicity, effects of changing refractive indices, as described in [11] are not taken into account here. For the reconstruction distance \( z \) from the camera sensor, a constant refractive index is used. Therefore, the reconstructed volume will be slightly compressed in the depth dimension \( z \). The complex field in distance \( z \) to the diffracting aperture, i.e. to the hologram, is thus approximated by

\[
C(x, y, z) = \frac{z}{i\lambda} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(\psi, \eta) e^{i2\pi r/\lambda} \frac{r}{p^2} \, d\psi \, d\eta
\]

where \( r \) is defined by \( r = \sqrt{(x - \psi)^2 + (y - \eta)^2 + z^2} \), \((x, y, z)\) describe the coordinates in the reconstructed plane in distance \( z \) to the diffracting hologram \( H \), and \( U \) denotes the field amplitude in the diffracting plane [6]. \( \lambda \) is the wavelength of the employed laser light.

For hologram reconstruction \( U(\psi, \eta) \) is replaced by the hologram intensity distribution superposed with a plane wave of normalized intensity 1. In favour of computational efficiency, the calculation is executed by three Fourier transformations. This yields, for each reconstructed distance \( z \) to the hologram plane, a slice of the complex object field

\[
C(x, y, z) = F^{-1} \left\{ F \left[ H(x, y) \cdot e^{-i2\pi c/\lambda} \right] \times F[k(x, y, z)] \right\} \quad \text{with} \quad k(z) = \frac{z}{i\lambda} \cdot e^{i2\pi \sqrt{x^2+y^2+z^2}/\lambda} \left(\frac{x^2+y^2+z^2}{\lambda}\right),
\]

while \( F \) refers to a two dimensional Fourier transformation and \( \times \) to a pointwise product. The reconstruction step is computationally expensive but also highly amenable to parallelisation. Hence, the calculation is executed by the graphics processing unit. Additionally, the region of interest in the holograms is set to the size of 1024 px \( \times \) 1024 px. Hence, the number of pixels in each dimension is a power of 2, which facilitates the speed of the fast Fourier transform algorithm.

4 Particle Image Simulation

Undoubtedly, it is not possible to calculate the true original object wave by executing equation 3 on a hologram which is roughly sampled by a digital sensor and has a comparatively limited extension due to the limited sensor size. In order to write a particle detection algorithm, one has to understand what a reconstructed particle image will look like.

As a first step, it is of particular interest, what the effect of a small transverse particle movement will be. In 2009, “most of the development of algorithms to reconstruct digital holograms is targeted to finding small
particles (typically 2 pixels widths [... in extent])" [12]. The system used here deals with even smaller particles of less than a single pixel in extent. To the authors’ knowledge, up to today, the algorithm development concentrates on particle diameters exceeding the pixel pitch of the recording sensors. It can be expected that for the small particles used in this investigation, already a small movement in the plane parallel to the camera sensor will affect the reconstructed particle image. To analyse this effect, the diffraction pattern generated by a circular disc of the diameter corresponding to the average particle diameter in the experimental system of 8.69 \( \mu m \) is simulated. By using Babinet’s principle and Fresnel-Kirchhoff diffraction theory, the complex wave in the distance of 30 cm is calculated. This means that the simulated particle distance corresponds to a particle which is approximately in the lateral centre of the cuvette in the experimental system shown in figure 2.1. The resulting complex object wave is superimposed with a plane reference wave. The retrieved intensity distribution is the simulated hologram of a particle on the optical axis and in distance of 30 cm to the sensor.

As visualized in figure 4.1, the diffraction field in the size of the camera sensor of \((1.228 cm)^2\) is simulated. The central circle with the diameter of the camera sensor of this simulated particle hologram is used to simulate particles in different locations in a constant distance of 30 cm to the sensor. This is done by moving the centre of the particle hologram to different locations on the simulated camera sensor. In the following, the coordinates in the plane perpendicular to the optical axis will be referred to by Euclidean coordinates \((x, y)\) with \(x\) describing the horizontal coordinate. Hence, the \((x, y)\) position of the simulated particle is moved from the optical axis in the centre of the simulated hologram to the desired transverse position. The spatial extension of the pixel matrix corresponding to the camera sensor area remains constant. Therefore, like in the optical experiment, a part of the diffraction pattern is deleted as it is cut off at the edges of the camera sensor.

![Fig. 4.1 Particle hologram simulation from a diffracting circular aperture.](image1)

![Fig. 4.2 Reconstruction of a particle image from a simulated particle hologram.](image2)

In order to tailor the hologram to the experimental conditions, the intensity is quantised to the bit depth of the recording camera. Additionally, the transverse resolution of the matrix is reduced to \(1024 \times 1024\) px\(^2\). This means that each pixel corresponds to \((12 \mu m)^2\), which is the pixel size of the camera used in the experiment.

Finally, similar to the reconstruction of the optically recorded holograms, the complex object wave is reconstructed by simulating the diffraction of a plane wave by the simulated hologram (see section 3). This is sketched in figure 4.2, where a plane wave illuminating the simulated hologram from the left yields five reconstructed intensity planes with the intensity maximum in the distance of 30 cm to the hologram. In the same way, also the phase values are retrieved. Therefore, the reconstructed particle images are comprised of several slices parallel to the hologram plane, reconstructed in predefined distances to the hologram. The distance of the slices is set to 50 \( \mu m \).
5 Particle Images Depending on the Subpixel Location

Particle images are simulated on a grid distributed over the whole camera sensor. As an example, four positions will be discussed here in more detail. These positions are marked by blue circles in figure 5.1 and are defined in pixel coordinates by \((x, y) \in \{(519.5, 519.5), (519.5, 526), (526, 519.5), (526, 526)\}\). Hence, one particle is simulated in the centre of four pixels, two particles are located in the centre between two pixels and one is simulated in the centre of a single pixel. The origin of the pixel coordinates is defined in the upper left corner of the \((x,y)\)-plane. Therefore, the centre of the pixel grid is located at \((x, y) = (512.5, 512.5)\)px.

Fig. 5.1 Transverse positions of particles simulated in a distance of 30cm to the camera sensor on a pixel grid with origin in the upper left corner of the 1024 × 1024 pixel matrix.

The intensity distributions in the focal plane of the simulated particle images are shown in figure 5.2. All intensity values are normalized to the reconstructed global maximum intensity of all four particle images. The intensity of the reconstructed particle image with \(x\) and \(y\) position in the centre of pixel \((526, 526)\) is more than twice as large as the intensity reconstructed in each of the pixels surrounding \((x, y) = (519.5, 519.5)\)px in the corresponding image shown in the upper left of figure 5.2. As the diffraction efficiency can be expected to be lower for particle holograms further away from the centre of the camera sensor at \((x, y) = (512.5, 512.5)\)px, this huge difference can be attributed to the location of the particle in relation to the pixel grid.

Figure 5.3 shows the gradation of the intensity parallel to the optical axis for a constant \((x, y)\)-position for each of the four simulated particles. Characteristics of the intensity fields of reconstructed particle images can be seen. In the ideal simulated case in the centre of a pixel position, the intensity in the focal plane reaches a distinct maximum while its value can clearly break down, if the location of a pixel edge is in the centre of the \((x,y)\)-position of the scattering particle. This is one reason, why the algorithm developed in the course of this work does not rely on intensity thresholds only for particle detection and deletion of ghost particles.

Figures 5.4 and 5.5 show intensity as well as phase planes through the same simulated particle images as in the previous two figures 5.2 and 5.3. This time, vertical planes parallel to the optical \(z\)-axis are shown. The intensity distributions show the expected behaviour of a comparatively well-defined transverse localisation of the intensity maximum, spreading over 2 pixel at maximum while the depth of focus is in the order of magnitude
of $mm$. In figure 5.5, the wrapped phase values are shown. These are the coloured representations of the phase values modulo $2\pi$. These values, on the other hand, show a very similar behaviour of contracting cones which expand again with opposite sign after having passed the focal plane. For a particle on the optical axis, this was simulated and measured for particles on a glass plate in [3]. This noticeable behaviour of the reconstructed phase values justifies the approach that the phase information should be employed for particle position detection as well as for the distinction between ghost particles and true particle images. Unfortunately, searching for highly symmetric phase cones and intensity maxima is not sufficient for a particle position detection and validation routine, as will become clear in the next section.

6 Particle Images Depending on Their Distance to the Borders of the Camera Chip

In a first step, the quality of the simulation is compared with results from the optical experiments. In an optical experiment, a particle was detected in position $(x,y) = (511, 519)\, \text{px}$. The resulting wrapped phase and normalized intensity values can be seen in figures 6.1 and 6.2 while the respective simulation results are shown in figures 6.3 and 6.4. The very good agreement between experiment and simulation can be seen at once. Hence, the particle simulation algorithm is sufficient to generate particle images which can be expected from the experimental system.

A slight fluctuation of the phase values close to the axis through the investigated $(x,y)$-position of the particle in the numerical simulation, which is not detectable in the experimental measurement, is attributed to the numerical truncation error. The second difference between experiment and simulation is that the intensity in the experiment seems to be more elongated in the depth direction, which means that the depth of focus is larger. This is explained by three effects. At first, the resolved number of interference fringes is probably more limited in the optical experiment, as many particles are present and the background shown in figure 2.2 additionally covers the interference fringes of particles. Therefore, the numerical aperture, which is directly related to the number of interference fringes recorded from the particle, is reduced. This directly results in an increased depth of focus, as can be seen in equation 1. Secondly, the experimental conditions are always accompanied by noise, for example due to the layers with different refractive indices in the beam path. Finally, which is probably the most important reason, the normalization is different. The reconstructed intensity spreads slightly more in the $(x,y)$-plane and in $z$ in the optical experiment than in the simulation, as expected from the reduced numerical aperture and the noise. Additionally, the $(x,y)$-position of the particle generating the recorded interference pattern is probably not precisely located in the centre of a pixel on the camera pixel grid. Therefore, the reconstructed intensity maximum is spread over more pixels than in the simulation. So the global maximum of the intensity values reconstructed in the particle image decreases. The intensity values displayed in figures 6.1 and the following are normalized to the global intensity maximum of the respective reconstructed particle image. When this maximum is decreased, smaller intensity values are enhanced in the respective image.
Apart from the described minor deviations between the experimental and the numerical data, a good agreement between the images is clearly visible. Symmetric contracting phase cones with changing sign in the focal plane of the particle and subsequent expansion can be measured as well as simulated. The intensity is elongated in the $z$-direction and comparatively concentrated in the $(x,y)$-position. Simply searching for a phase jump for particle localisation will not work. The reason is that, due to the phase wrapping, phase discontinuities can be found in the whole reconstructed image volume. Therefore, from these images, a rough estimation of the particle location by searching for intensity maxima and a consecutive comparison with expected particle images is promising.

Closer to the edges of the camera sensor, e.g. surrounding position $(x,y) = (427,86)\text{px}$, images are reconstructed as shown in the figures 6.5 and 6.6. The phase cone and the intensity distribution in the $yz$-plane at $x = 427\text{px}$ appear distorted while the $xz$-plane at $y = 86\text{px}$ is remaining comparatively symmetric apart from the experimental background noise. In order to answer the question whether such a structure can be attributed to a real particle in the measurement volume, the particle image expected from the same position $(x,y) = (427,86)\text{px}$ was simulated with the algorithm paraphrased in section 4. The resulting images are shown in figures 6.7 and 6.8. Again, a good agreement between simulation and experimental results can be seen and confirms that the reconstructed image was caused by a scattering particle closer to the upper edge of the hologram. Therefore, a particle search algorithm also has to be able to detect particle images being distorted in such a way.

The reason for the particle image distortion in the $yz$-plane can be found in the shorter distance of the particle to the sensor edge. Therefore, more diffraction fringes of the particle are cut off by the sensor edge and cannot contribute to the formation of the reconstructed particle image. Hence, in the dimension in which many fringes are lost, the particle image is reconstructed in a distorted way. In summary, the decisive factor for the shape of a particle image is the location of the respective particle in the plane parallel to the recording camera sensor, perpendicular to the optical axis.

### 7 An Adapted Particle Search Algorithm

The previous sections show that the shapes of particle images vary distinctly depending on their location in the plane parallel to the sensor plane of the recording camera. The objective of a particle detection algorithm is the detection of particle positions from the reconstructed object field in the measurement volume. For this purpose, an algorithm is developed which correlates the intensity as well as the phase values expected from a particle image in a specific location in the plane perpendicular to the optical axis, i.e. in a defined $(x,y)$-location.

A rough estimation of the particle location from local intensity maxima yields a set of suspected particle positions with $(x,y,z)$ coordinates. For each of these maxima, four reference patterns in phase and intensity are

Fig. 6.1 $xz$-planes of experimentally reconstructed wrapped phase values and normalized intensity values of a particle close to the centre of the sensor [13].

Fig. 6.2 $yz$-planes of experimentally reconstructed wrapped phase values and normalized intensity values of a particle close to the centre of the sensor [13].
Fig. 6.3. $xz$-planes of simulated wrapped phase values and normalized intensity values of a particle close to the centre of the sensor [13].

Fig. 6.4. $yz$-planes of simulated wrapped phase values and normalized intensity values of a particle close to the centre of the sensor [13].

Fig. 6.5. $xz$-planes of experimentally reconstructed wrapped phase values and normalized intensity values of a particle close to the upper sensor edge [13].

Fig. 6.6. $yz$-planes of experimentally reconstructed wrapped phase values and normalized intensity values of a particle close to the upper sensor edge [13].

Fig. 6.7. $xz$-planes of simulated wrapped phase values and normalized intensity values of a particle close to the upper sensor edge [13].

Fig. 6.8. $yz$-planes of simulated wrapped phase values and normalized intensity values of a particle close to the upper sensor edge [13].
generated for the respective estimated \((x, y)\) location. Four patterns are generated due to the clearly differing particle images depending on the subpixel location (see section 5). So, the reference particle image with the maximum correlation value within a predefined surrounding area of the estimated particle position is selected. A particle position is finally retrieved as the position of this global correlation maximum which additionally exceeds a predefined correlation threshold.

As the particle simulation algorithm is computationally expensive, about 26000 reference images are calculated and stored as a look-up table before the particle detection algorithm is executed. Based on these reference patterns, for each estimated particle position, four reference patterns are generated by superposition. These four reference patterns, one for each subpixel location, are used for the particle detection around each estimated position.

The holograms of 92 time steps resolved with \(620\text{Hz}\) are treated by the algorithm. The experimental setup is shown in figure 2.1. The stirrer is located approximately in the centre of the cuvette. The cuvette is located with an offset of about \(1.4\text{cm}\) to the optical axis of the system such that lower \(x\) values in the reconstruction correspond to the area closer to the cuvette wall. The upper border of the recording volume is located below the stirrer. An approximately circular flow is expected with a higher velocity at higher \(x\)-values.

The retrieved particle positions are tracked with the tracking code published by [14] based on the algorithm described in [15]. For particles being attributed to a single trajectory the maximum movement of a particle between two images is set to \(800\mu\text{m}\) while the \(xy\)-positions are rescaled before the tracking in order to account for the expected higher flow velocity in the \(z\) direction. After the tracking, the rescaling is undone. The minimum trace length of 10 timesteps and a maximum number of lost timesteps in a single trajectory is set to 4. Finally, all detected particle positions attributed to a single trace are fitted by a polynomial and the root mean square error between these particle positions and their respective traces are calculated. For a total of 114 traces resulting from 1668 tracked particle positions the maximum deviation of all positions are listed in table 7.1.

In table 7.1 the expected smaller deviation between the particle positions and the polynomial fits in the \(x\) and \(y\) coordinates perpendicular to the optical axis can be seen. Still, the deviation of the longitudinal position \(z\) is smaller than the approximated depth of focus in the optical system of more than \(1\text{mm}\). Additionally, figure 7.1 shows the projection of the particle traces to the \(xz\)-plane. The reconstructed volume within the cuvette is seen from below. Already after the recorded time of 0.15s, the expected circular flow behaviour becomes apparent. Figure 7.2 shows the reconstructed three dimensional volume.

<table>
<thead>
<tr>
<th>dimension</th>
<th>maximum deviation between a particle position and its respective trace / (\mu\text{m})</th>
<th>root mean square error of the detrended data of all reconstructed traces / (\mu\text{m})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>106</td>
<td>17</td>
</tr>
<tr>
<td>(y)</td>
<td>74</td>
<td>12</td>
</tr>
<tr>
<td>(z)</td>
<td>1059</td>
<td>260</td>
</tr>
</tbody>
</table>

Table 7.1 Maximum deviation and root mean square error calculated from all 1668 tracked particle positions to their respective 114 traces retrieved by fitting the traces with polynomials.

8 Summary and Outlook

It is demonstrated in numerical simulations and in experiments that particle images reconstructed from digital holograms can vary distinctly for the same particle. Changes of the image appearance can be attributed to the location of the particle during the recording process. Therefore, a different reconstructed particle image will be retrieved from a holographic recording of a particle if its position changes in the plane perpendicular to the optical axis. If this position corresponds to a pixel edge on the camera chip or a pixel centre, the reconstructed image will differ in its global intensity maximum and the shape of the reconstructed phase and intensity images. Therefore, a subpixel resolution can only be achieved, if this is taken into account in the
particle position detection algorithm. Particles of the same size are not reconstructed with the same intensity maximum. Algorithms using intensity thresholds for particle detection should take this effect into account. Furthermore, the truncation error due to the loss of diffraction fringes on the sides of a camera sensor is very prominent and hence distorts the reconstructed particle images.

Based on this experience, an algorithm is developed, which also detects particles close to the sensor edges. The main idea of the algorithm is that a different reconstructed particle image is to be expected when the measured particle moves perpendicularly to the optical axis. Therefore, in the correlation analysis executed in the algorithm, different reference images are used for each estimated particle location for the fine resolution. The expected particle images are simulated and used as reference patterns in a particle detection algorithm. This algorithm relies on the phase and the intensity values of the reconstructed sample volume and the simulated particle images. Correlation values between these patterns are used for particle position detection and validation in an optical experiment. Finally, particle traces are retrieved from the reconstructed particle positions. In a first application of the algorithm, the average root mean square error of these traces is clearly below the previously estimated depth of focus for the imaging system. Future research will deal with the appropriate threshold of the correlation values to distinguish ghost from real particles and the application of the developed algorithm to longer time series.

9 Acknowledgements

The authors thank you for having shown interest in our research by reading this text and the Niedersächsische Ministerium für Wissenschaft und Kultur (Ministry for Science and Culture of Lower Saxony) for financial support.

References


